18. Prove that if T in $A(V)$ has all its characteristic roots in F, then there is a basis of V in which the matrix of T is triangular.

19. Prove that the elements S and T in $A(V)$are similar in $A(V)$ if and only if they have the same elementary divisors.

20. Prove that the linear transformation T on V is unitary if and only if it takes an orthonormal basis of V into an orthonormal basis of V.

NOVEMBER/DECEMBER 2023

23PMA11 — ALGEBRAIC STRUCTURES

Time : Three hours Maximum : 75 marks

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SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define conjugate relation on a group G.

2. List out the conjugate classes of the group S_3.

3. Define solvable group.

4. Define the R-module of a ring.

5. Give an example for a linear transformation on R^2.

6. Define the cyclic subspace of a vector space.

7. Define basic Jordan block.

8. Define the characteristic polynomial of a linear transformation.

9. What do you mean by unitary transformation?

10. Define normal transformation.

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) Prove that normalizer of an element is a subgroup of G.

Or

(b) State and prove the second part of Sylow's theorem

12. (a) Prove that if G is a solvable group and if G' is a homomorphic image of G, then G' is solvable.

Or

(b) Prove that if A and B are groups. then A × B is isomorphic to B × A.

13. (a) Prove that the relation of similarity is an equivalence relation in $A(V)$, where $A(V)$ is the set of all vector space homomorphisms of V into itself.

Or

(b) Prove that there exists a subspace W of V. invariant under T, such that $V = V_1 \oplus W$.

14. (a) Prove that the minimal polynomial for T over F is the least common multiple of $p_1(x)$ and $p_2(x)$, where $p_1(x)$, $p_2(x)$ are the minimal polynomials of linear transformations T_1 and T_2 respectively over F.

Or

(b) Suppose that two matrices A, B in F_n are similar in K_n, where K is an extension of F. Prove that A and B are already similar in F_n.

15. (a) If A is invertible, prove that $tr(ACA^{-1}) = tr(C)$.

Or

(b) If T in $A(V)$ is such that $(vT, v) = 0$ for all v in V, then prove that $T = 0$.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. State and prove Sylow's theorem.

17. Prove that every finite Abelian group is the direct product of cyclic groups.